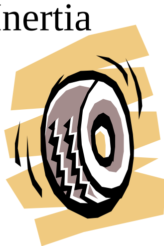
Chapter 9

Mass Moment of Inertia



Miami University

1

Definition

- Mass Moment of Inertia:
  - –Resistance to Rotational Acceleration

2

Review: Moment of Inertia

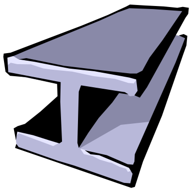
- Definition: Sum of each incremental strip of area times its distance from the centroidal axis.

$$I_x = y^2 \Delta A$$

3

Moment of Inertia

- The physical shape of a beam can influence its resistance to bending
- Areas at a greater distance increase the moment of inertia



$$I_x = y^2 \Delta A$$

4

Moment of Composites

- The *parallel axis theorem* must be used when areas are shifted away from the centroidal axis.

$$I_x = I_c + Ad^2$$

$$where\ I_c = Formula\ from\ Table$$

$$A = area$$

$$d = dist.\ between\ axes$$

5

Radius of Gyration

- Definition: The theoretical distance from the axis where the entire area could be concentrated and still have the same moment of inertia

$$k = \sqrt{\frac{I_c}{A}} \quad or \quad I_c = Ak^2$$

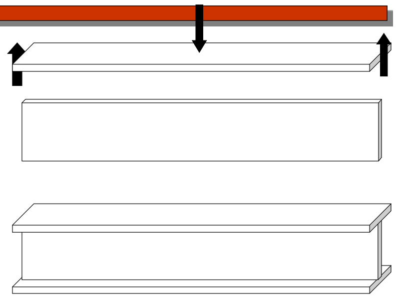
6

Simple Beams

Weak

Stronger

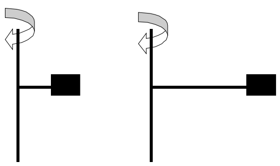
Strongest



Dynamics – Mass in Motion

Mass Moment of Inertia

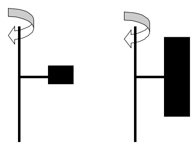
- Mass Moment of Inertia is a function of the mass of a rotating object and its distance from its axis



Mass Moment of Inertia

$$I = \Sigma r^2 \Delta m$$

$$where\ r = radius,\ m = mass$$



Units

- Metric: Kg-m^2
- English: slug-ft^2 or lb-ft-s^2

$$I = \Sigma r^2 \Delta m = r^2 \times \frac{W}{g}$$

$$I = ft^2 \times \frac{lb_f}{32.2\frac{ft}{s^2}} = lb \cdot ft \cdot s^2$$

Formulae for *I*

- Page 319, Table 9-2
- Note a different ***I*** for each axis
- Be careful in identifying the axis of rotation
- Use combinations of formulae for composite bodies

Sample Problem #1

- Twirling a baton (long column) that is: 1.0" dia x 3.0 ft long made of solid aluminum (.098 lb/cu. in.)

$$I_z = \frac{1}{12} m\left(3r^2 + l^2\right)$$

13

Calculate Weight of Bar

$$Vol = \frac{\pi\ D^2}{4} \times l = \frac{\pi\left(1^2\right)}{4} \times \left(3 \times 12\right)$$

$$Weight = Vol \times \left(0.098 \frac{lb}{in^3}\right)$$

$$Weight = 2.77\ lb$$

14

Determine Mass of Bar

$$m = \frac{W}{g} = \frac{2.77\,lb}{32.2\,\frac{ft}{s^2}} = 0.086\,\frac{lb \cdot s^2}{ft}$$

15

Calculate Mass Moment

$$I_z = \frac{1}{12} m\left(3r^2 + l^2\right)$$

$$I_z = \frac{1}{12}(0.086)\left[\left(3 \times \left(\frac{0.5}{12}\right)^2\right) + 3^2\right]$$

$$I_z = 0.065\,lb \cdot ft \cdot s^2$$

16

Mass Moment of Composites

- Use combinations of formulae for composite bodies
- Remember that bodies NOT on the axis of rotation must be transferred using a *parallel axis theorem.*
- Total Mass moment of inertia is the sum of the individual mass moments

17

Mass Parallel Axis Theorem

$$I = I_c + md^2$$

$$where\ I_c = Formula\ from\,Table$$

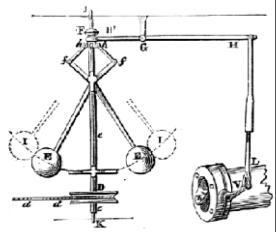
$$m = mass$$

$$d = dist.\ between\ axes$$

18

Sample Problem #2

- The governor on old steam engines used a pair of spinning balls to regulate rpm. The change in radius of the balls changes its mass moment of inertia.



19

Steam Governor

- The balls are 3” in dia, made of cast iron and weigh 3.2 lbs each. The centers of the balls are 1.0 ft apart. Neglect the mass of the connecting rod.
- Calculate the mass moment of inertia.

20

Determine moment of each ball

$$I_c = \frac{2}{5} m r^2$$

$$I_c = \frac{2}{5} \times \frac{3.22\, lb}{32.2 \frac{ft}{s^2}} \times \left(\frac{1.5}{12}\ ft \right)^2$$

$$I_c = 0.000625\ lb \cdot ft \cdot s^2$$

21

Determine moment at 6” radius

$$I = I_c + md^2$$

$$I = 0.000625 + \left[\left(\frac{3.22\, lb}{32.2 \frac{ft}{s^2}} \right) (0.5\, ft)^2 \right]$$

$$I = 0.025625$$

22

Sum the Mass Moments

- Add the mass moments of inertia of both balls together (connection rod was neglected).
- 0.0256 + 0.0256 = 0.0512 lb-ft-s^2

23

Sample Problem #3

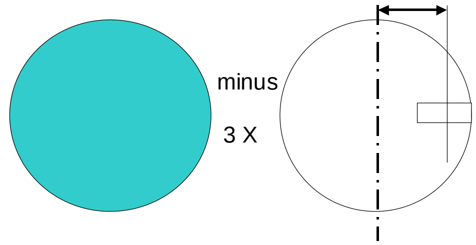
- Composite bodies can also be used to represent holes in the object.
- Determine the mass moment of inertia first as if the object were solid.
- Then *subtract* the mass moment of the absent area.

24

Sample Problem #3

- Determine the mass moment of inertia of a standard bowling ball.
- Calculate the mass moment of a solid ball first.
- Then calculate the mass moment of the three finger holes, using the density of the ball material, at the correct distance from the center of the ball.

Bowling Ball Moment



Radius of Gyration of Bodies

- Similar to 2 dimensional system, but mass is used instead of area
- k represents a linear dimension (either meters or feet)

$$k = \sqrt{\frac{I}{m}} \quad or \quad I = k^2 m$$

Typical Examples

- Inertia in flywheels to size motor
- Minimizing moments in race cars to make them turn “easier”
- Sizing small stepper motors to control robotic arms
- Determining acceptable “spin up” time on generators and motors

Homework

- Read Chapter 10
- Chapter 9 Problems
  - #45, 47 & 50